## Memoryless Property of the Geometric Random Variable

What do we mean when we say that a random process has no memory? Let's try to specify mathematically what takes place when a process continually "starts over".

Recall a few preliminaries. For the geometric series we have the $n^{th}$ partial sum given as follows:

$$\sum_{k=0}^{n} a^k = \frac{1-a^{(n+1)}}{1-a}$$

For example,

$$\sum_{k=0}^{3} \left(\frac{1}{2}\right)^k = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{1-\frac{1}{2}^{(3+1)}}{1-\frac{1}{2}} = \frac{15}{8}$$

For a geometric random variable where we count the number of failures till the first success on a sequence of independent Bernoulli random variables we have the probability mass function

$$Prob(Y=k) = (1-p)^k\, p$$

To consider the probability that the number of trials will be more than some number, say $s$, we calculate the "survival function"

$$Prob(Y > s) = 1 - Prob(Y \leq s) = 1 - F(s)$$

$$Prob(Y > s) = 1 - \sum_{k=0}^{s} (1-p)^k\, p = 1 - p\left[\frac{1-(1-p)^{(s+1)}}{1-(1-p)}\right] = (1-p)^{(s+1)}$$

Now suppose that a certain number of trials (failures!) have already gone by, say $t$ of them. What is the probability it will now take more than $s$ failures till the first success? We evidently want

$$P(Y > s+t \mid Y \geq t) = \frac{P([Y > s+t] \cap [Y \geq t])}{P(Y \geq t)} = \frac{P(Y > s+t)}{P(Y \geq t)}$$

$$P(Y > s+t \mid Y \geq t) = \frac{(1-p)^{(s+t+1)}}{(1-p)^{(t)}} = (1-p)^{(s+1)} = P(Y > s)$$

That is, knowing that we've already seen $t$ failures doesn't change the likelihood that it will take at least $s$ more trials till the first success. We aren't "due" a success in any way!